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Spin fluctuation theory of nearly ferromagnetic metals

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Abstract. Based on the spin fluctuation theory, we propose a way to analyse the magnetic properties of nearly ferromagnetic metals by introducing quantities corresponding to the saturation magnetization in the ground state and the Curie temperature for weak ferromagnets. We also show how to evaluate the spin fluctuation parameters from experiments. As an example, magnetic properties of paramagnetic $Y(Co_{1-x}Al_x)_2$ are discussed.

1. Introduction

The magnetic properties of weakly ferro- and antiferromagnetic materials are determined by the nature of spin fluctuations contained in them. This observation has been verified by quantitative comparisons of the theoretical predictions with experiments. For insulator magnets, the magnetic properties are well described by the Heisenberg model which is characterized by several exchange interaction constants. Likewise, the self-consistently renormalized spin fluctuation theory explains various magnetic properties of weak ferromagnets in terms of the parameters characterizing the spin fluctuation spectrum (Moriya 1979, 1985). Therefore, if enough experiments are done, we can determine the spin fluctuation spectrum of the system and with the use of the parameters thus determined we can compare the theoretical predictions with experiments (Takahashi and Moriya 1984, Lonzarich 1984). Such efforts have been made on a number of weak ferromagnets by direct measurements of the spin fluctuation spectrum by NMR relaxation measurements (Yoshimura *et al* 1987) or from magnetic measurements (Shimizu *et al* 1990, Nakabayashi *et al* 1992). Most of the analyses so far have been confined to the case of weakly ferromagnetic metals. The reason is that the analysis is clearcut in this case and much experimental data are available from the magnetic measurements, for example, the Curie temperature T_c, the saturation magnetization σ_s in the ground state and the slope of the Arrott plot.

In the case of exchange-enhanced paramagnets, i.e. nearly ferromagnetic systems, such an analysis has not been done, although the system is very close to weak ferromagnetism and therefore nearly the same theoretical framework is expected to apply. Konno and Moriya (1987), in their treatment of the specific heat of nearly ferromagnetic metals, analysed paramagnetic Ni_3Ga. Their analysis, however, was slightly different from the one used for weakly ferromagnetic cases. For example, they introduced a temperature scale T^* as a quantity corresponding to the critical temperature T_c for ferromagnets. We will show in the following that another definition of T^* is also possible, more closely related to the ferromagnetic Curie temperature T_c.

In the present paper, we present a more unified description of the magnetic properties of nearly ferromagnetic metals in a way parallel to that of weak ferromagnets by extending our previous approach (Takahashi 1986, 1990, 1992) dealing with the spin fluctuation

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effects of weakly ferromagnetic metals. As noted above, the reason a large number of quantitative comparisons between theory and experiment have been made came from the fact that parameters characterizing the magnetic properties are easily obtained experimentally for weak ferromagnets. On the other hand, we have no parameters corresponding to the saturation magnetization in the ground state and the Curie temperature in the case of exchange-enhanced paramagnets. We often do not have enough parameters for quantitative comparison, and so we therefore introduce parameters which correspond to these quantities for nearly ferromagnetic materials which are determined experimentally. The purpose of the present paper is to show that we can describe the magnetic properties of nearly ferromagnetic metals in almost the same way as for weak ferromagnets. We also derive the relation connecting the microscopic spin fluctuation parameters with the observable magnetic properties and propose a way to estimate the spin fluctuation parameters from experiments, thus giving us a method for quantitative comparison between theory and experiment.

The plan of the paper is as follows. In the next section, we give a short theoretical description of how to deal with the exchange-enhanced paramagnets by simply extending our previous work on weak ferromagnets. Based on the results, we show how to analyse experimental data in the following section. The final section contains discussion of our results.

2. Formalism

The following discussions are implicitly based on the model Hamiltonian for conduction electrons with a strong intra-atomic electron–electron Coulomb repulsion as the origin of the magnetism like the Hubbard Hamiltonian. Rather than manipulating the model, we employ the phenomenological approach. Our starting point is to assume that the squared local spin amplitude $\langle S^2 \rangle$ is almost independent either of the temperature or of the external field variation, as has been indicated experimentally (Ziebeck *et al* 1982, Shiga *et al* 1988). Contrary to insulator magnets, this may not be a strict statement, but we assume it is still valid in the temperature range of interest, which is supposed to come from strong electron–electron correlation effects. It is based on the consideration that we need high-energy excitations compared to the low-lying magnetic excitations in order to change the total spin fluctuation amplitude (Takahashi 1986).

With the use of the fluctuation-dissipation theorem from statistical mechanics, the equal-time spin correlation function is represented in terms of the dynamical spin susceptibility $\chi^{\alpha\beta}(q, \omega)$ as follows:

$$\langle \{\delta S_q^\alpha, \delta S_{-q}^\beta\}_+ \rangle = 2 \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \coth(\omega/2kT) \mathrm{Im}\chi^{\alpha\beta}(q, \omega) \tag{1}$$

where $\{A, B\}_+$ represents the anti-commutation relation between operators A and B, δS_q^α is the α component of the spin deviation operator $S_q^\alpha - \langle S_q^\alpha \rangle$ of wave vector q, and $\langle \cdots \rangle$ represents the thermal average. In the paramagnetic phase, by decomposing the factor $\coth(\omega/2kT)$ into the bose factor $n(\omega)$ and a constant term, the almost constant total amplitude $\langle S^2 \rangle$ can be given as a sum of the zero-point and thermal spin fluctuation parts,

$$\langle S^2 \rangle = \langle S^2 \rangle_{\mathrm{zp}} + \langle S^2 \rangle_{\mathrm{th}}. \tag{2}$$

We define each component in equation (2) in terms of the dynamical spin susceptibility $\chi(q,\omega)$ ($=\chi^{\alpha\alpha}(q,\omega)$, $\alpha = x$, y and z) as follows:

$$\langle S^2\rangle_{zp} = \frac{3}{N_0^2}\sum_q \int_0^\infty \frac{d\omega}{\pi}\mathrm{Im}\chi(q,\omega)$$

$$\langle S^2\rangle_{th} = \frac{6}{N_0^2}\sum_q \int_0^\infty \frac{d\omega}{\pi} n(\omega)\mathrm{Im}\chi(q,\omega) \tag{3}$$

$$n(\omega) = (e^{\omega/kT}-1)^{-1}$$

where k is the Boltzmann constant and N_0 is the number of magnetic ions in the crystal. Note that in equation (2) we used the term *zero-point* fluctuations in an extended sense. Our definition agrees with the usual meaning only in the ground state. It does depend on temperature through $\chi(q,\omega)$. In the case of weak ferromagnets, the low-energy part of the spin fluctuation spectrum is known to be well described by the following Lorentzian form (Ishikawa *et al* 1985):

$$\mathrm{Im}\chi(q,\omega) = \frac{\chi}{1+q^2/\kappa^2}\frac{\omega\Gamma_q}{\omega^2+\Gamma_q^2} \qquad \Gamma_q = \Gamma_0 q(\kappa^2+q^2) \tag{4}$$

where χ represents the static uniform magnetic susceptibility in units of $(g\mu_B)^2$ and κ is the inverse of the temperature-dependent correlation length. The random phase approximation applied to the Hubbard model gives $\chi(q,\omega)$ in the form of equation (4) in the small-(q, ω region. The gyromagnetic ratio g is assumed to be 2 hereafter. In the following, we assume that $\chi(q,\omega)$ is highly enhanced in the small-(q, ω) region, and we employ the same form of the spectrum (4). According to our previous treatments on weak ferromagnets, we characterize the q, ω behaviour of $\chi(q,\omega)$ in equation (4) by introducing the following energy scales kT_0 and kT_A:

$$kT_0 = \Gamma_0 q_B^3/2\pi \qquad kT_A = (q_B^2/2\kappa^2)(N_0/\chi) \tag{5}$$

where q_B is the effective zone boundary wave vector corresponding to the unit volume v_0 per magnetic ion defined by $(6\pi^2/v_0)^{1/3}$. These parameters T_0 and T_A give the measure of the width of distribution of $\mathrm{Im}\chi(q,\omega)/\omega$ in q, ω space. We further introduce the dimensionless reciprocal susceptibility y by

$$y = \frac{1}{2kT_A}\frac{N_0}{\chi}. \tag{6}$$

Then in terms of the parameters defined above, $\mathrm{Im}\chi(q,\omega)$ is rewritten as follows:

$$\mathrm{Im}\chi(q,\omega) = \frac{N_0}{2kT_A}\frac{1}{x^2+y}\frac{\omega[2\pi kT_0 x(y+x^2)]}{\omega^2+[2\pi kT_0 x(y+x^2)]^2} \tag{7}$$

where x is the reduced wave vector defined by $x = q/q_B$.

From our definition (3) it follows that the zero-point amplitude $\langle S^2\rangle_{zp}$ is dependent on temperature through y. On the other hand, the thermal part $\langle S^2\rangle_{th}$ depends both on y and the temperature T through the bose factor $n(\omega)$. By substituting equation (7) into equation (3) and integrating with respect to ω and q, we can express $\langle S^2\rangle_{zp}$ as a function of y. In the present paper, because we are particularly interested in the highly exchange-enhanced

paramagnetic cases, y is considered to be very small. Now by expanding $\langle S^2\rangle_{zp}(y)$ around the origin $y = 0$ with respect to y, we see that the y dependence of $\langle S^2\rangle_{zp}$ can be well expressed in the following y-linear form (Takahashi 1986):

$$\langle S^2\rangle_{zp}(y) = \langle S^2\rangle_{zp}(0) - (9T_0/2T_A)y + \cdots. \tag{8}$$

In the same way, after integrating equation (3) with respect to ω, $\langle S^2\rangle_{th}$ is written in the following form:

$$\langle S^2\rangle_{th} = \frac{9T_0}{T_A}\int_0^1 dx\, x^3\{\log(U) - 1/2U - \psi(U)\}$$
$$U = T_0x(y + x^2)/T \tag{9}$$

where we used the definition of the digamma function $\psi(z)$:

$$2\int_0^\infty \frac{t\,dt}{(t^2+z^2)(e^{2\pi t}-1)} = \ln z - 1/2z - \psi(z).$$

As was discussed by Takahashi (1986), weakly ferromagnetic materials are characterized by their large zero-point amplitude $\langle S^2\rangle_{zp}(0)$, almost comparable to the total amplitude. In the case of weak ferromagnets, the size of $\langle S^2\rangle_{zp}(0)$ is assumed to be slightly smaller than $\langle S^2\rangle$. In the present case of exchange-enhanced paramagnets, let us assume $\langle S^2\rangle_{zp}(0)$ to be slightly larger than $\langle S^2\rangle$, i.e.

$$\langle S^2\rangle_{zp}(0) > \langle S^2\rangle. \tag{10}$$

Then with the use of equations (2) and (8), it follows that

$$(9T_0/2T_A)y > \langle S^2\rangle_{th} \geqslant 0.$$

This means that if condition (10) is satisfied, y becomes finite through all the temperature range down to $T = 0$ K and no magnetic transition occurs.

Because the thermal spin fluctuation amplitude vanishes in the ground state, the above temperature-independent slight difference between $\langle S^2\rangle$ and $\langle S^2\rangle_{zp}(0)$ can be well characterized in terms of the small ground-state value of y as follows:

$$\langle S^2\rangle_{zp}(0) - \langle S^2\rangle = (9T_0/2T_A)y_0 \tag{11}$$

where y_0 stands for the ground-state value of y. Then substituting equations (8), (9) and (11) into equation (2), we get the following equation for y:

$$y = y_0 + 2\int_0^1 dx\, x^3\{\log(U) - 1/2U - \psi(U)\}$$
$$U = T_0x(y + x^2)/T. \tag{12}$$

We can utilize equation (12) to find the temperature dependence of the magnetic susceptibility χ, since y is proportional to the inverse of χ.

Now we introduce a temperature scale T^* from the condition $y = 2y_0$ at $T = T^*$. By putting $y = 2y_0$, $T = T^*$ and $x = \eta z$ in equation (12), we obtain the equation determining T^* from y_0:

$$y_0 = 2\eta^4 \int_0^{1/\eta} dz\, z^3 \{\log(u) - 1/2u - \psi(u)\}$$
$$u = z(y_0/\eta^2 + z^2) \tag{13}$$

where we introduced a parameter η by $\eta = (T^*/T_0)^{1/3}$. We see from the above condition the physical meaning of T^*. Since y is proportional to the inverse of χ, T^* is defined as the temperature where the magnetic susceptibility is half its ground-state value. If we take into account the Curie–Weiss behaviour of the magnetic susceptibility in a wide temperature range, theoretically predicted and observed experimentally, the above condition roughly amounts to assuming that $\chi \propto 1/(T + T^*)$, in contrast with $\chi \propto 1/(T - T_c)$ for weak ferromagnets. Therefore T^* defined here is the natural extension of T_c. From equation (13) we can easily see that y_0 is almost proportional to η^4 and is given by

$$y_0 \simeq 2c\eta^4$$
$$c = \int_0^{\infty} dz\, z^3 \{\log(z^3) - 1/2z^3 - \psi(z^3)\} \tag{14}$$

for small η.

In the presence of the static uniform magnetization σ (per magnetic ion in units of the Bohr magneton μ_B) under an externally applied magnetic field H along the z axis, equation (2) has to be slightly modified. The total squared amplitude $\langle S^2 \rangle$ is now represented by the sum of the squared uniform magnetization, and two components of squared transverse and longitudinal fluctuation amplitudes as follows:

$$\langle S^2 \rangle = \tfrac{1}{4}\sigma^2 + \langle S_x^2 \rangle + \langle S_y^2 \rangle + \langle \delta S_z^2 \rangle \tag{15}$$

where δS_z represents the deviation of S_z from the static uniform component. In the presence of uniform magnetization, the magnetic susceptibility becomes anisotropic, and therefore the zero-point spin fluctuation amplitudes also become anisotropic as given by

$$\langle \delta S_z^2 \rangle_{zp}(y) = \tfrac{1}{3}\langle S^2 \rangle_{zp}(0) - \frac{3T_0}{2T_A} y_z + \cdots$$
$$\langle S_x^2 \rangle_{zp}(y) = \langle S_x^2 \rangle_{zp}(y) = \tfrac{1}{3}\langle S^2 \rangle_{zp}(0) - \frac{3T_0}{2T_A} y + \cdots \tag{16}$$

where y and y_z are introduced as the dimensionless transverse and the longitudinal susceptibilities defined by

$$y = \frac{1}{kT_A}\frac{h}{\sigma} \qquad y_z = \frac{1}{kT_A}\frac{\partial h}{\partial \sigma} \qquad h = g\mu_B H. \tag{17}$$

Of course, the thermal spin fluctuation amplitudes also become anisotropic reflecting the anisotropic susceptibilities. In the ground state, the thermal amplitude vanishes in

equation (15). Therefore at $T = 0$ K by substituting equation (16) into equation (15) and with the use of equation (11) we obtain

$$\frac{T_A}{9T_0}\frac{\sigma^2}{4} - \frac{1}{6}(y_z + 2y) = -\frac{y_0}{2}$$

$$y_z = \sigma \partial y/\partial \sigma + y. \tag{18}$$

The second line of equation (18) simply comes from the definition of equation (17). Equation (18) is a direct extension of equation (12) in the presence of the finite magnetization σ at $T = 0$ K. Mathematically it is regarded as a first-order differential equation of y with respect to σ. The magnetization process at $T = 0$ K is thus determined by solving y with respect to σ and the solution is easily found to be

$$y = y_0 + \frac{2T_A}{15T_0}\frac{\sigma^2}{4} = \frac{1}{kT_A}\frac{h}{\sigma}. \tag{19}$$

Note that in the case of weak ferromagnets, a negative constant term appears on the right-hand side of the first line of equation (19) and $h = 0$ gives a finite magnetization σ in the ground state. On the other hand, in the present case, because of the positive first term, no finite magnetization appears in the vanishing-h limit. Let us here introduce the third-order expansion coefficient $\bar{F}_1$ of h in the ground state with respect to σ (i.e. the fourth-order expansion coefficient of the free energy) by

$$h = kT_A y_0 \sigma + \tfrac{1}{8}\bar{F}_1 \sigma^3. \tag{20}$$

Then, from equation (19), the coefficient $\bar{F}_1$ is given by

$$\bar{F}_1 = \frac{4kT_A^2}{15T_0}. \tag{21}$$

From equation (19) we see that good linearity holds in the Arrott plot (σ^2 against h/σ) in the ground state. By solving σ^2 with respect to h/σ in equation (19), we obtain

$$\frac{\sigma^2}{4} = \frac{15T_0}{2kT_A^2}\frac{h}{\sigma} - \frac{15T_0}{2T_A}y_0. \tag{22}$$

In the same way as for the weakly ferromagnetic cases, let us introduce a new parameter $(\sigma^*)^2$ by extrapolating the Arrott plot to the $h/\sigma = 0$ limit, as shown in figure 1 schematically. From equation (22) we see $(\sigma^*)^2$ is given by

$$\frac{(\sigma^*)^2}{4} = \frac{15T_0}{2T_A}y_0 \simeq \frac{15T_0}{T_A}c\left(\frac{T^*}{T_0}\right)^{4/3} \tag{23}$$

where we have used the approximate relation $y_0 \simeq 2c(T^*/T_0)^{4/3}$, equation (14), valid for small η. It is interesting to note that equation (23) has the same form as the one connecting

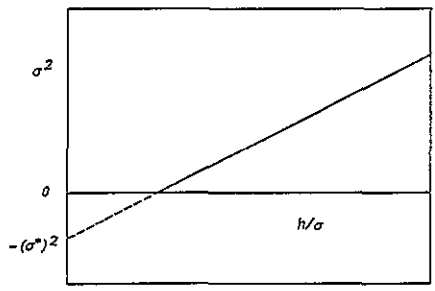

Figure 1. Schematic Arrott plot showing how to estimate the σ^* value.

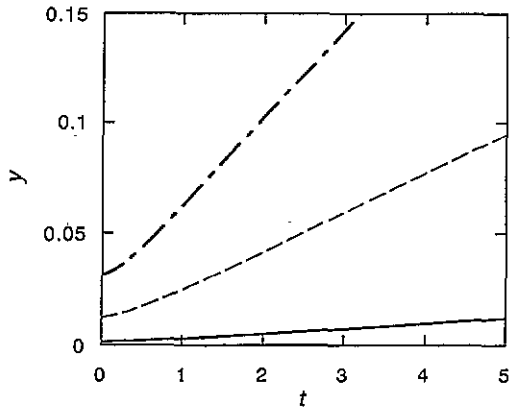

Figure 2. t dependence of y for $T^*/T_0 = 0.1$ (full curve), 0.2 (broken curve) and 0.3 (chain curve).

T_c and σ_s for weak ferromagnets. Therefore, we could successfully introduce the parameter σ^* which plays the same role as the saturation magnetization σ_s.

Finally, for the temperature dependence of the magnetic susceptibility, equation (12) can be put in the following dimensionless form by introducing the reduced temperature t:

$$y = y_0 + 2\eta^4 \int_0^{1/\eta} dz\, z^3 \{\log(u) - 1/2u - \psi(u)\}$$
$$u = z(y/\eta^2 + z^2)/t \qquad t = T/T^*. \tag{24}$$

By solving equation (24) numerically, we see that a good Curie–Weiss-like t-linear temperature dependence is produced in a wide temperature range, except for low temperatures where t^2 behaviour is observed, as shown in figure 2. In particular, note that the t dependence of y is determined by the single parameter η since y_0 is uniquely determined by η. Moreover, because η is related to the value T^*/T_0, the slope of the y–t curve (i.e. the value of dy/dt) is determined by the single parameter T^*/T_0. The Curie–Weiss law for magnetic susceptibility reduces to the following form in the present dimensionless treatment:

$$\frac{dy}{dt} \simeq \frac{6T^*}{T_A\sigma_{eff}^2} = \frac{\eta^3}{5y_0}\left(\frac{\sigma^*}{\sigma_{eff}}\right)^2 \simeq \frac{1}{10c\eta}\left(\frac{\sigma^*}{\sigma_{eff}}\right)^2 \tag{25}$$

where we have used the first line of equation (23) and the definition of η in the second line of equation (25). Therefore, to be consistent with equation (25), the above results indicate that the ratio σ_{eff}/σ^* has to be determined by the single parameter η, i.e. the ratio of T^*/T_0. From the same argument, Takahashi (1986) proposed that the ratio σ_{eff}/σ_s should be determined by the T_c/T_0 value for weak ferromagnets. Equation (25) shows that the same relation also holds between σ_{eff}/σ^* and T^*/T_0.

3. Analysis of experiments

In the preceding section, we introduced the parameters σ^* and T^* as quantities corresponding to the saturation magnetization σ_s at $T = 0$ K and the Curie temperature T_c in the case of

weak ferromagnets, respectively. These quantities, as well as $\bar{F}_1$ and χ_0, are estimated by magnetic measurements. All the above quantities are, of course, not independent of each other. For instance, the following relation holds:

$$\tfrac{1}{4}(\sigma^*)^2\bar{F}_1 = N_0/\chi_0. \tag{26}$$

In order to estimate the values of T_0 and T_A directly, we will need NMR relaxation measurements or neutron scattering experiments.

Let us first consider the case that the value of T_0 is already known from experiment. Then by substituting the following definition of y_0:

$$y_0 = \frac{1}{2kT_A}\frac{N_0}{\chi_0} \tag{27}$$

into equation (23), we obtain the following result for T_A:

$$T_A^2 = \frac{15T_0}{k(\sigma^*)^2}\frac{N_0}{\chi_0} = 15\bar{F}_1T_0/4. \tag{28}$$

Putting the above expression for T_A into equation (27) again, the following expression for y_0 is obtained:

$$y_0 = \sigma^*\left(\frac{1}{60kT_0}\frac{N_0}{\chi_0}\right)^{1/2} = \frac{(\sigma^*)^2}{4}\left(\frac{\bar{F}_1}{15kT_0}\right)^{1/2}. \tag{29}$$

From the above results, if we can estimate T_0 from NMR relaxation experiments, for example, then T_A and y_0 can be evaluated with the use of equations (28) and (29) in terms of χ_0, σ^* and $\bar{F}_1$ obtained by magnetic measurements. The value of T^* is then estimated by

$$T^*/T_0 = \eta^3 = (y_0/2c)^{3/4}. \tag{30}$$

Even if we know neither T_0 nor T_A, we can estimate both of them. For example, by eliminating y_0 from equations (21) and (22) we get an expression for T_0:

$$T_0 = T^*(2c/\sigma^*)^{6/5}(60kT^*\chi_0/N_0)^{3/5}. \tag{31}$$

The value of T_A is then obtained by using equation (28). However, we need reliable estimates for T^* and χ_0. Theoretically, the parameter T^* is a well-defined quantity, i.e. the temperature giving half of the ground-state susceptibility value. Unfortunately, it is often very difficult to extract a reliable estimate for T^* from the observed temperature dependence of the susceptibility measurement, for reasons discussed below. Therefore it may not be practical to use equation (31) in most cases.

As an example of the above prescription for the analysis of experiments, let us discuss the experimentally observed magnetic properties of the paramagnet $Y(Co_{0.89}Al_{0.11})_2$ (Yoshimura *et al* 1988). From NMR relaxation measurements, T_0 is estimated to be 932 and 1534 K depending on the observed nucleus, ^{59}Co and ^{27}Al, respectively. From the extrapolation of the Arrott plot at 4.2 K, we obtain $\sigma^* = 0.064$. It is difficult to estimate the static uniform magnetic susceptibility at $T = 0$ K, since its temperature dependence shows a broad maximum with a value 7.5×10^{-5} emu/g around $T_m = 40$ K (Sakakibara *et al* 1990). Instead of using the χ_0 value, let us estimate T_A and y_0 from the observed slope of the

Table 1. Spin fluctuation parameters for $Y(Co_{0.89}Al_{0.11})_2$ estimated from experiments.

T_0 (K)	T_A (K)	y_0	T^*/T_0	σ_{eff}/σ^*
932	1.3×10^4	0.0019	0.012	25
1534	1.7×10^4	0.0015	0.010	29

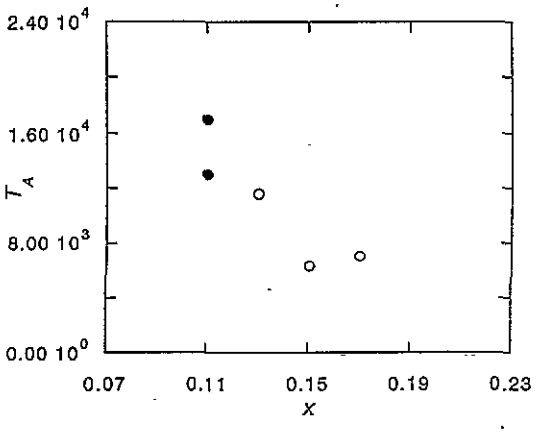

Figure 3. Estimated T_A values of $Y(Co_{1-x}Al_x)_2$ alloys. Open circles represent values for the ferromagnetic case, full circles represent the present estimates.

Arrott plot, i.e. the value $\bar{F}_1/k = 4.82 \times 10^4$ K. The results are shown in table 1. We show in figure 3 our estimated T_A values along with those obtained for ferromagnetic alloys (Yoshimura *et al* 1987). From the figure we see that T_A increases from the ferromagnetic side towards the paramagnetic region. Our estimated paramagnetic values are in agreement with this trend. The estimated ratios T^*/T_0 are also shown in the table. With the use of the T^*/T_0 value shown in table 1, we evaluated the value of σ_{eff}/σ^* from the slope of the calculated temperature dependence of $y(t)$. The results are also shown in table 1. Experimentally, σ_{eff} is estimated to be $2.54\mu_B$, resulting in the ratio $\sigma_{eff}/\sigma^* = 40$, which is in rather good agreement with our theoretical estimates.

If we estimate the χ_0 value from $\bar{F}_1$ and σ^* using equation (26), we obtain $\chi_0/N_0 = 2.0 \times 10^{-2}$ K, almost three times larger than the observed maximum susceptibility value around $T_m = 40$ K. From table 1 we see that T^* is about 20 K lower than T_m, only half the value of T_m. We suppose that below T_m the susceptibility is suppressed, for reasons not taken into account in the present treatment. Therefore a reliable estimate of T^* from the susceptibility measurement is difficult in the present case.

4. Discussion

The present work is a simple extension of our previous works on weakly ferromagnetic metals. We have shown that we can treat exchange-enhanced paramagnets in the same theoretical framework as the weakly ferromagnetic case. In particular, we have shown that we can define quantities σ^* and T^* which correspond to the saturation magnetization σ_s in the ground state and the Curie temperature T_c for weak ferromagnets, respectively. Then all the relations for weak ferromagnets also hold in the present case if we replace σ_s and T_c by σ^* and T^*, respectively. This is very important because it gives us a way of analysing the magnetic properties of exchange-enhanced paramagnets quantitatively associated with the microscopic spin fluctuation spectrum of the materials.

As mentioned in the introduction, Konno and Moriya (1987) have already considered nearly ferromagnetic metals. They introduced a parameter K_0, and from K_0 a temperature scale T^* defined by $T^* = T_0 K_0^3$. Our y_0 value corresponds to K_0^2 in their notation. In order to estimate K_0 and T^*, they argued that they had to know both T_0 and T_A values. Therefore for the analysis of Ni_3Ga, they had to use the T_0 and T_A values for the isoelectronic compound Ni_3Al. Otherwise they utilized the enhancement factor of the linear specific heat coefficient to estimate K_0^2. One of the main differences with the present work lies in our relation of $\bar{F}_1$ in terms of T_A and T_0, i.e. equation (21) which comes from our underlying assumption of the constant total spin fluctuation amplitude. Because of the relation, the number of independent parameters is reduced, which makes it easy to extract the spin fluctuation parameters from experiments.

As was discussed in the preceding section, it is often difficult to estimate T^* from the observed temperature dependence of the magnetic susceptibility because of the ambiguity of the χ_0 value in the ground state. The susceptibility generally consists of various contributions, such as the diamagnetism from the closed ion cores, the orbital contributions, and the spin part of the valence electrons, etc. It is not easy to extract the absolute value of the electron spin contribution. Moreover, the susceptibility is affected by various inhomogeneities at low temperature. Most importantly for nearly ferromagnetic metals, metamagnetic behaviour is often observed by applying an external magnetic field. Associated with the metamagnetism, the susceptibility shows a maximum in its temperature dependence (Sakakibara *et al* 1990). We suppose that the susceptibility is suppressed at low temperature from some unknown interactions which are related to the origin of the metamagnetism. This makes it difficult to find a reliable susceptibility value for the hypothetical substance showing no metamagnetism at low temperature. In the present theoretical framework we can explain neither the metamagnetic behaviour nor the susceptibility maximum at low temperature. We can only derive a monotonic decrease of the susceptibility with increasing temperature. We therefore suppose that we need some extra mechanisms for a proper description of the metamagnetic behaviours, which is out of the scope of the present treatment.

There are many compounds which show ferromagnetism if we replace a constituent element of an alloy with another one like the YCo_2 discussed in the present paper. Almost all the experimental analyses have focused on the ferromagnetic sides. We hope that experimental effort will also be expended on the paramagnetic region and the results tested according to the present prescription.

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